① $V_1 = 1\,l = 10^{-3}\,m^3$

$V_2 = 0{,}05\,l = 5 \cdot 10^{-5}\,m^3$

$p_1 = 105\,kPa = 105\,000\,Pa$

$\varkappa = 1{,}5$

$pV^{\varkappa} = konšt.$

$$W_{ex} = -\int_{V_1}^{V_2} p\,dV \qquad (0{,}5)$$

$$p_1 V_1^{\varkappa} = pV^{\varkappa} \qquad (1\text{b})$$

$$p = p_1 V_1^{\varkappa} V^{-\varkappa}$$

$$W_{ex} = -\int_{V_1}^{V_2} p_1 V_1^{\varkappa} V^{-\varkappa}\,dV = -p_1 V_1^{\varkappa} \int_{V_1}^{V_2} V^{-\varkappa}\,dV = -p_1 V_1^{\varkappa} \left[\frac{V^{-\varkappa+1}}{-\varkappa+1}\right]_{V_1}^{V_2} =$$

$$= \frac{-p_1 V_1^{\varkappa}}{1-\varkappa}\left[V_2^{1-\varkappa} - V_1^{1-\varkappa}\right] \qquad (0{,}5\text{b})$$

$$= \frac{-105\,000\,(10^{-3})^{1{,}5}}{1-1{,}5}\left[(5\cdot 10^{-5})^{1-1{,}5} - (10^{-3})^{1-1{,}5}\right] =$$

$$= \frac{105\,000\,(10^{-3})^{1{,}5}}{0{,}5}\left[(5\cdot 10^{-5})^{-0{,}5} - (10^{-3})^{-0{,}5}\right] = \frac{105\,000}{0{,}5}\,10^{-1{,}5}\left(\frac{1}{\sqrt{5\cdot 10^{-5}}} - \frac{1}{\sqrt{10^{-3}}}\right)$$

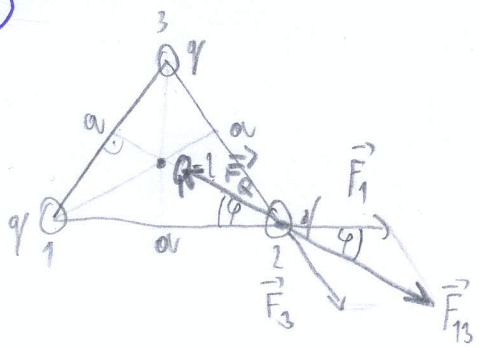
$$= 729{,}15\ J \text{ - spätná} \qquad (1\text{b})$$

$$\Delta S = \int \frac{dQ}{T} \;;\; dQ = 0 \;\Rightarrow\; \Delta Q = 0$$

$$\Delta S = 0\ JK^{-1} \qquad (1\text{b})$$

4b

(2)



$$|\vec{F}_1| = |\vec{F}_3|$$

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$$

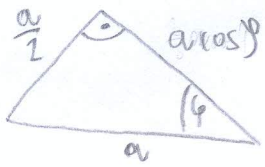
$$\varphi = 30°$$

$$F_{13} = 2F_1\cos\varphi = 2\,\frac{1}{2\cdot 4\pi\varepsilon_0}\frac{q^2}{a^2}\cos 30°$$

$$F_{13} = \frac{1}{2\pi\varepsilon_0}\frac{q^2}{a^2}\frac{\sqrt{3}}{2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\sqrt{3}$$ **1b**

$$F_Q = F_{13}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{\left(\frac{2}{3}a\cos\varphi\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\sqrt{3}$$ **1b**



$$\frac{Q}{\frac{4}{3}\cdot\frac{3}{4}} = \sqrt{3}\,q$$

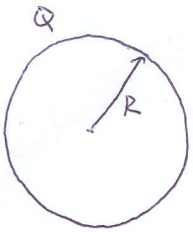
$$3Q = \sqrt{3}\,q$$

$$|Q| = \frac{q}{\sqrt{3}} \rightarrow Q = -\frac{q}{\sqrt{3}}$$ **1b**

$$Q = -\frac{0{,}3\,\mu C}{\sqrt{3}} = -\frac{3}{10\sqrt{3}}\,\mu C = -\frac{\sqrt{3}}{10}\,\mu C$$

3b

3.



$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$Q(r) = \rho \frac{4}{3}\pi r^3 = \frac{Q}{\frac{4}{3}\pi R^3} \frac{4}{3}\pi r^3 = \frac{Q r^3}{R^3}$$

$r < R$

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q(r)}{\varepsilon_0} \quad (E \parallel dS)$$

$$\oint E \cdot dS = \frac{Q r^3}{R^3 \varepsilon_0}$$

$$E \oint dS = \frac{Q r^3}{R^3 \varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q r^3}{R^3 \varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi \varepsilon_0 R^3} r} \quad (1b)$$

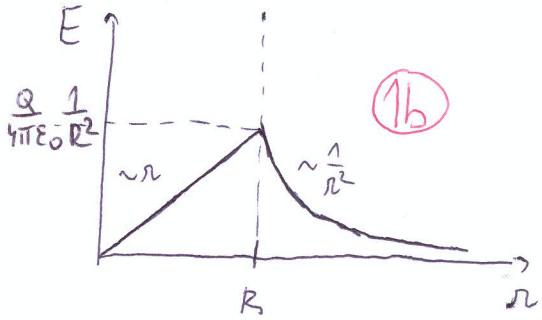
$r > R$

$$\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$E \oint dS = \frac{Q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi \varepsilon_0} \cdot \frac{1}{r^2}} \quad (1b)$$



(1b)

(3b)

4 PETER BALLO

(0,5b) (0,5b)

1b